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Strange particles freeze-out conditions from a new hadronic equation of state

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Abstract. We propose a thermodynamically consistent equation of state (EOS) for a hot and dense handron gas (HG) incorporating the hard-core finite-size correction for baryons which result in a Van-der Waals excluded volume effect. We develop a complete description within a thermal hadron gas model to determine the production of strange particles in relativistic nucleus-nucleus collisions and calculate the strange particle ratios as function of baryon chemical potential μ_B and temperature T. The results are compared with other existing thermodynamically consistent EOS. Furthermore we extract the thermal freeze-out parameters for different strange particles using the latest data obtained by the CERN experiments.

1 Introduction

High-energy heavy-ion collisions offer the possibility of creating a high-density nuclear matter in the initial stage of the collision. We believe that a colour deconfined matter may be produced for a very brief time, such a matter is called as a quark-gluon plasma (QGP). Recently many theoretical and experimental papers have appeared on the properties and signals of QGP [1–4]. Strangeness production has been a much studied possible signature for the identification of QGP formation in relativistic nucleus-nucleus collisions [5–8]. It has been suggested that in the baryon-rich QGP phase, there would be an enhanced strange quark production followed by its speedy equilibration which will result in a large equilibrium strangeness density. However, models based on purely hadronic scenario also show an enhancement of strangeness. In order to have a reliable prediction regarding strangeness enhancement, knowledge of proper equation of state (EOS) for a hot and dense hadron gas (HG) is essential. An ideal gas approximation for HG with point-like baryons and mesons has been found unsuitable because the point-like baryons can be compressed to an infinitely large density. Moreover, it has also been demonstrated that without a repulsive interaction in the hadronic EOS, one cannot construct a first order quark-hadron phase transition using Gibbs' criteria in the whole temperature T-chemical potential μ plane. A phenomenological solution to this difficulty has been suggested by introducing a short range and repulsive hard-core interactions between a pair of baryons or antibaryons in the form of an excluded volume effect [9–14] which is akin to the well-known Van-der-Waals method in molecular physics. However, we notice that the original suggestion of Cleymans et al. [9] as well as of Kuono and Takagi [11] were not thermodynamically consistent because the thermodynamical quantities cannot be derived from a thermodynamical potential or a partition function. Some recent approaches [13, 14] have taken into consideration the effect of finite-size correction in a thermodynamically consistent manner. However, these methods involved cumbersome solutions and numerical evaluations of various thermodynamical quantities pose a problem. Here we propose a thermodynamically consistent equation of state (EOS). We will attempt to calculate the various strange particle ratios as a function of baryon chemical potential μ_B and temperature T and the results are compared with the existing thermodynamically consistent EOS [13, 14]. Finally we will extract the thermal freeze-out parameters for various hadrons using CERN experimental data obtained by NA35 [15] and others [16, 17].

One of the central problems in the study of high energy heavy-ion collisions lies in deducing the state of the system by determining temperature and chemical potential formed in the process from the observed final particle properties. The freeze-out stage of the system when the particles fly towards the detectors without further interactions, is directly connected to the observed multiplicity distributions of the particles. We first connect the thermodynamical properties of the system to an appropriate EOS for the hot and dense hadron gas. Finally we deduce the freeze-out conditions of the thermal HG fireball formed in the heavy-ion collisions. Recently many attempts have been made to determine the freeze-out conditions of the fireball [18–21]. On the other hand strangeness enhancement has also been explained in the conventional hadronic quark gluon string model and relativistic quantum molecular dynamics (RQMD) type of transport approach [22–23] where thermalized fireball concept has completely been ignored. It is indeed encouraging to notice that simple thermal models [18–21] can explain the data on the strangeness enhancement. However, the whole exercise crucially depends on the EOS used to determine the thermal and chemical parameters at the freeze-out of the fireball of a hot and dense HG. The purpose of this paper is to obtain a new thermodynamically consistent EOS and compare its predictions for the freeze-out conditions with those of the other approaches.

2 Formulation of the model

The HG models describe the hadronic matter as a gas of hadrons with different masses where individual mass states are populated according to equilibrium Bose-Einstein or Fermi-Dirac distributions. This is formulated within the framework of grand canonical ensemble by introducing baryon and strange chemical potentials μ_B and μ_S, which are required to guarantee the baryon number and strangeness conservation in the strongly interacting hadronic matter. When applying these HG models to situations of high temperatures and large baryon chemical potentials, one must introduce a short range, hard-core repulsive interaction between a pair of baryons or antibaryons. The repulsive part has usually been parametrized in phenomenological models by assigning the hadrons a geometric hard-core volume which results into an excluded volume effect [9–11]. However, the main drawback of these models is their lack of consistency with the basic thermodynamical relations obtainable from a partition function. Rischke et al. [13] suggested a novel method for restoring the thermodynamical consistency by replacing the chemical potential $\tilde{\mu}_j = \mu_j - V_j p^{\text{ex}}(T, \mu_j)$ where $p^{\text{ex}}(T, \mu_j)$ is the pressure of the HG after excluded volume correction, and V_j is the eigenvolume of the j^{th} type of hadrons, we finally get a transcendental solution for $p^{\text{ex}}(T, \mu_j)$ as:

$$p^{\text{ex}}(T, \mu_j) = \Sigma_j p_j^0(T, \mu_j - V_j p^{\text{ex}}) \tag{1}$$

where $p_j^o(T, \mu)$ is the pressure due to j^{th} type of hydrons with modified chemical potential. Although the method of incorporating the excluded volume effect in Rischke model rests on a mathematically sound technique, it still lacks a clear physical interpretation. Moreover, the transcendental solution one gets from Rischke model makes the numerical evaluation of the problem more difficult.

Recently Uddin and Singh [14] proposed a thermodynamically consistent model in which the grand canonical partition function Z is modified as follows:

$$\begin{aligned}\ln Z_i = \; & (g_i/6\pi^2 T)\int_{V_i}^{V-\Sigma_j N_j V_j} dV' \int_0^\infty \left\{k^4/(k^2+m_i^2)^{1/2}\right\} \\ & \times[1/\{\lambda_i^{-1}\exp(E_i/T)+1\}]dk\end{aligned} \tag{2}$$

Here V is the total volume of the system, g_i is the spin-isospin degeneracy factor, m_i is the mass and λ_i is the fugacity for the i^{th} species of baryons, k is the momentum of hadron and N_j is the number of j^{th} type of baryons. Using Boltzmann approximation, we get:

$$\ln Z_i = V(1 - \Sigma_j n_j V_j) I_i \lambda_i \tag{3}$$

where

$$I_i = (g_i/2\pi^2) m_i^2 T k_2(m_i/T)$$

Using the expression for number density of i^{th} baryon species as

$$n_i = (\lambda_i/V)(\partial \ln Z_i/\partial\lambda_i)\big|_{T\cdot V'}$$

we get from (3)

$$n_i = (1 - \Sigma_j n_j V_j) I_i \lambda_i - I_i \lambda_i^2 \{\partial(\Sigma_j n_j V_j)/\partial\lambda_i\} \tag{4}$$

Defining $R = \Sigma_j n_j V_j$ which indicates the fraction of the occupied volume in the total volume of the system, we get a differential equation:

$$\begin{aligned}&\Sigma_j I_j V_j \lambda_j^2 (\partial R/\partial\lambda_j) \\ &\quad +(\Sigma_j I_j V_j \lambda_j + 1)R - \Sigma_j I_j V_j \lambda_j = 0\end{aligned} \tag{5}$$

Using the method of 'parametric line' one can define the fugacities in the parametric space as:

$$\lambda_j = 1/(a_j + I_j V_j t) \tag{6}$$

We finally get the solution of (5) as:

$$R = 1 - \frac{\int_t^\infty \{\exp(-t')/G(t')\}dt'}{\{\exp(-t)/G(t)\}} \tag{7}$$

where

$$G(t) = t\prod_{j=2}^{p}(a_j + I_j V_j t)$$

Here p gives the total number of baryon species. The parameter t is fixed by setting $a_1 = 0$ so that $t = 1/I_1 V_1 \lambda_1$. If one knows R, one can obtain $\partial R/\partial\lambda_i$ numerically. Obviously the solution (7) cannot be a unique one because one of the constants a_1 has been arbitrarily fixed as zero.

Instead of solving the differential equation (4), in the parametric space, one can get simple solution provided one substitutes $\partial(\Sigma_j n_j V_j)/\partial\lambda_i = (\partial n_i/\partial\lambda_i)V_i$. Here we assume that n_i depends only on the fugacity λ_i of the i^{th} species in the hadron gas consisting of multicomponent baryons.

Thus we get a modified differential equation as:

$$\begin{aligned}&\frac{\partial n_i}{\partial\lambda_i} + n_i\left\{\left(\frac{1}{I_i V_i \lambda_i^2}\right) + \left(\frac{1}{\lambda_i}\right)\right\} \\ &\quad = \left(\frac{1 - \Sigma_{j\neq i} n_j V_j}{V_i \lambda_i}\right)\end{aligned} \tag{8}$$

The solution of (8) can be obtained in a straight forward manner:

$$n_i = \frac{(1 - \Sigma_{j\neq i} n_j V_j) Q_i}{V_i \lambda_i \exp(-1/I_i V_i \lambda_i)} \tag{9}$$

where

$$Q_i = \int_0^{\lambda_i} \exp(-1/I_i V_i \lambda_i) d\lambda_i$$

In this model one can obtain R as follows:

$$R = \Sigma_j n_j V_j = X/(1+X) \tag{10}$$

where

$$X = \Sigma_j Q_j / \{\lambda_j \exp(-1/I_j V_j \lambda_j) - Q_j\}$$

One can interpret X as the ratio of occupied and the available volume. Finally the number density of i^{th} baryon species can be written as:

$$n_i = \{(1-R)/V_i)[Q_i/\{\lambda_i \exp(-1/I_i V_i \lambda_i) - Q_i\}] \quad (11)$$

It is obvious from (11) that we have obtained an easy and simple solution of (4). There are no parameters in the theory and thus it can be regarded as a unique solution. However, this still depends crucially on the assumption that the number density n_i of the i^{th} species of baryons is a function of the fugacity λ_i alone and it is completely independent of the fugacities of other kinds of baryons. Without this assumption one may be able to find other solution. Moreover, we notice that the calculation of all the thermodynamical quantities become easier in the present model.

The entire hadron gas spectrum upto the mass of 1400 MeV can be described in terms of two chemical potentials μ_B and μ_S which give baryon number and strangeness conservations, respectively. The two sets of chemical potentials for the quarks and hadrons are related by $\mu_B = 3\mu_q$ and $\mu_S = \mu_q - \mu_s$. In a strangeness neutral QGP fireball μ_s is always exactly zero and hence $\mu_S = \mu_B/3$. Since the net strangeness of the colliding nuclei in the initial state is zero, strangeness conservation dictates that the net strangeness of the fireball should remain close to zero. However, the strangeness distribution among hadrons of different masses modifies the strangeness chemical potential μ_S far different than $\mu_B/3$ where μ_B is the baryon chemical potential. The condition for vanishing total strangeness takes the form:

$$n_{\text{strange meson}} - n_{\text{antistrange meson}} + n_{\text{strange baryon}} - n_{\text{antistrange baryon}} = 0 \quad (12)$$

where n is the number density. The case of exact strangeness neutrality in a HG in absolute chemical equilibrium has been explored in detail by Cleymans and Satz [24].

3 Results and discussion

In the recent past several efforts [18–21] have been made to explain the experimental results in terms of the simple framework of a thermal fireball model. Such approaches have explained the strangeness abundances in terms of a few macroscopic parameters eg. temperature T, chemical potentials μ_B and μ_S etc. The values of these parameters at the freeze-out point yield a complete thermodynamical description of the ratios of the observed particles. In our model, we have used the multicomponent HG consisting of seven baryonic states and eight mesonic states as shown in Table 1. The proper volume of the baryons (antibaryons) is obtained by employing the bag model considerations. Taking the hard- core radius of a nucleon (antinucleon) as $r_N = 0.6$ fm and mass M_N as 940 MeV, we can get the bag constant B from $4B = M_N/V_N$ where V_N is the eigenvolume of a nucleon ($V_n = 4\pi r_N^3/3$). The proper volume of other baryons is obtained by using $V_i = M_i/4B$.

Table 1. List of particles and antiparticles used in the calculations

Particle	Mass (MeV)	Strangeness	Degeneracy factor (g_i)
π	140	0	3
K	490	1	2
η	548	0	1
ρ	770	0	9
ω	785	0	3
K^*	890	1	6
η'	958	0	1
ϕ	1015	0	3
N	940	0	4
Λ	1120	−1	2
Σ	1190	−1	6
Δ	1236	0	16
Ξ	1320	−2	4
Σ^*	1385	−1	12
Λ^*	1405	−1	2

Table 2. List of the experimental total particle multiplicities and the particle ratios in the full phase space for the central S-S collisions at 20 GeV/A [15] used for the determination of thermal freeze-out parameters

K^+	12.5	±0.4
K^-	6.9	±0.4
Λ	8.2	±0.9
$\bar{\Lambda}$	1.5	±0.4
$p - \bar{p}$	20	±3
h^-	98	±5
$\bar{\Lambda}/\Lambda$	0.18	±0.05
K^+/K^-	1.8	±0.1
$\Lambda/(p-\bar{p})$	0.41	±0.08

The main purpose of this paper is to propose a thermodynamically consistent equation of state for the HG and to compare its predictions with other existing models. We have therefore, shown the variations of the ratios $\bar{\Lambda}/\Lambda$, $\bar{\Xi}/\Xi$, $\bar{\Xi}/\bar{\Lambda}$, (Ξ/Λ), K^+/K^- and $\Lambda/(p-\bar{p})$ with respect to baryon chemical potential μ_B at $T = 200$ MeV in Figs. 1–5, respectively. We compare the results obtained in the present model with those from other thermodynamically consistent approaches, eg., Uddin-Singh model as well as Rischke model. We have also plotted the results from the point-particle model so that we can notice the difference arising due to the finite-size excluded volume effect on these ratios. As expected our results almost overlap on the curves obtained in the Uddin-Singh approach. This justifies the approximation made in getting (8). However, our predictions differ from those of Rischke model or the point particle model. We also notice that the differences in the predictions for $\bar{\Xi}/\bar{\Lambda}$ or $\Lambda/(p-\bar{p})$ in different models are more significant than those for $\bar{\Lambda}/\Lambda$ or $\bar{\Xi}/\Xi$. It arises because the repulsion introduced in the form of an excluded volume effect cancels out to some extent in the

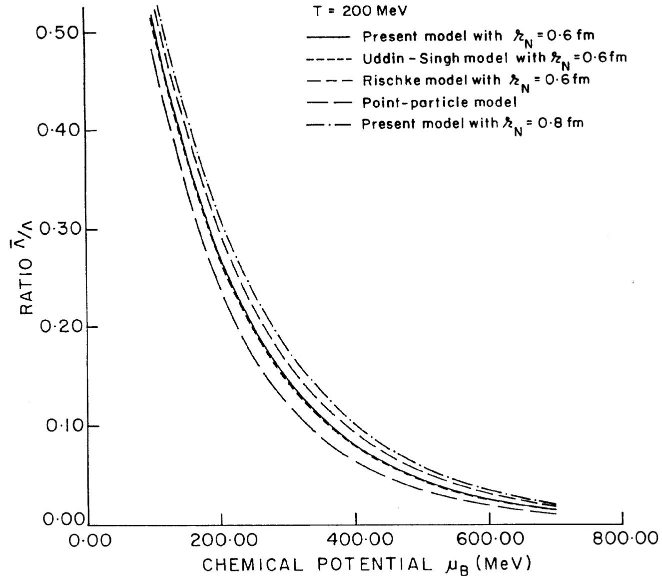

Fig. 1. The variation of $\bar{\Lambda}/\Lambda$ ratio with baryon chemical potential at temperature $T = 200$ MeV is calculated in point-like HG model and compared with present model as well as with Rischke and Uddin-Singh model

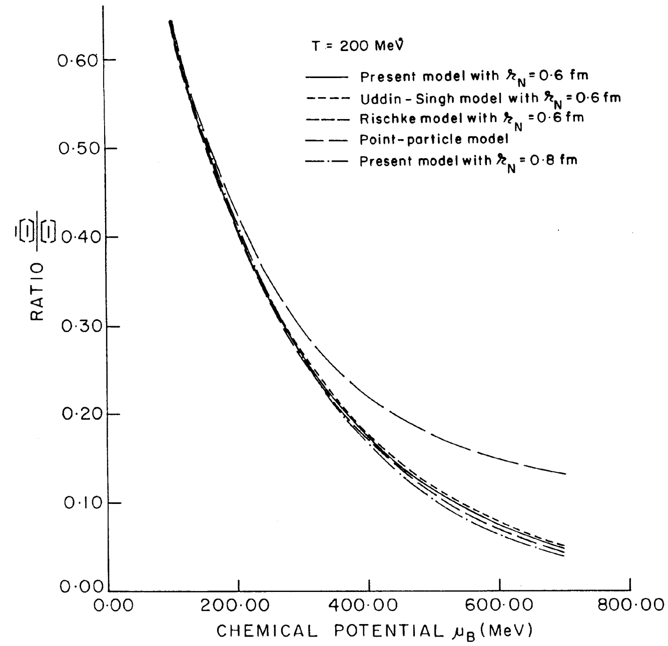

Fig. 2. The variation of $\bar{\Lambda}/\Xi$ ratio with baryon chemical potential at $T = 200$ MeV is compared in different HG models

ratio of $\bar{\Lambda}$ and Λ, or $\bar{\Xi}$ and Ξ. This explains why our result lies close to that of point-particle case in Figs. 1–2 for small values of μ_B. Figures 1–5 also reveal the effect of excluded volume correction as we find the curves for $\bar{\Xi}/\bar{\Lambda}$ and $\Lambda/(p-\bar{p})$ in the present model differ significantly from the point-particle case. Moreover, these ratios as obtained in the various thermodynamically consistent approaches, are considerably different at large μ_B. Thus Figs. 3 and 5 reveal the difference in the above models in a more obvious way. For comparison, we have also shown the results of our present model for the value of hard-core radius of nucleon $r_N = 0.8$ fm.

The experimental results by NA35 collaboration for full phase-space for S-S collisions at 200 GeV/A are $\bar{\Lambda}/\Lambda = 0.18 \pm 0.05$, $K^+/K^- = 1.8 \pm 0.1$ and $\Lambda/(p-\bar{p}) = 0.41 \pm 0.08$. Similarly WA85 results for mid-rapidity S-W collisions in the p_T interval $1.2 < p_T < 3.0$ GeV [17] at the same energy are $\bar{\Xi}/\Xi = 0.45 \pm 0.05$, $\bar{\Xi}/\bar{\Lambda} = 0.21 \pm 0.02$ and $\Xi/\Lambda = 0.095 \pm 0.006$. In order to get a fit to NA 35 data, we use a new parameter and we call it as strangeness saturation factor γ_s $(0 < \gamma_s \leq 1)$ which allows us to parametrize the incomplete chemical equilibration of the strange particles so that all the ratios can be fitted simultaneously for the same values of μ_B and T [18]. We can then define the effective fugacity of each strange quark as $\gamma_s \lambda_s$ and that of strange antiquark as $\gamma_s \lambda_s^{-1}$. Since the value of μ_s is exactly zero for a QGP fireball, we except γ_s to be equal to one. However, in the conventional HG picture, $\gamma_s < 1$ as one gets in p-p case [18, 25]. A full strangeness saturation $\gamma_s = 1$ in the HG picture would reveal a surprisingly short strangeness equilibration time-scale in S-S collisions.

It is worth emphasizing here that the values of the ratios $\bar{\Xi}/\Xi$ and $\bar{\Lambda}/\Lambda$ depend mostly on the relation of μ_S with μ_B at a fixed value of temperature. This relation is obtained by using the strangeness conservation con-

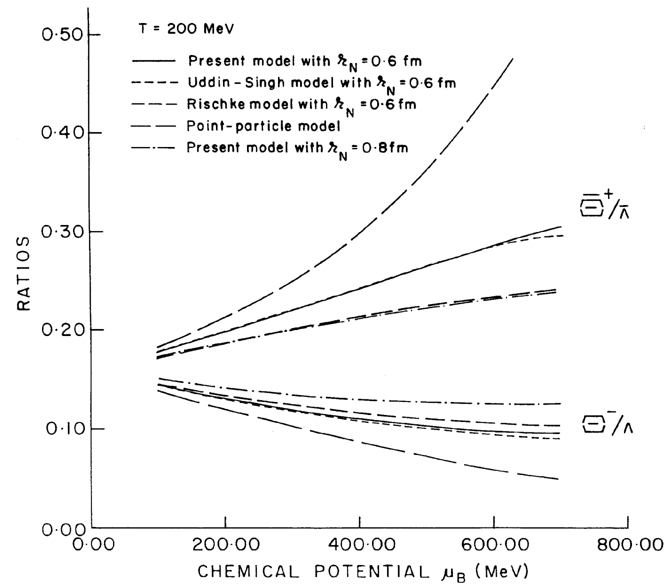

Fig. 3. The variation of ratios $\bar{\Xi}^+/\bar{\Lambda}$ and Ξ^-/Λ with baryon chemical potential at $T = 200$ MeV is compared in different HG models

straints in hadron gas model. For the HG resulting from the hadronization of QGP, we get $\mu_S = \mu_B/3$. However, for a HG phase without any QGP formation, the strangeness is distributed among mesons and baryons of much different masses, and one usually gets a completely different relation. This is a significant point because it avoids misinterpretation when evaluating the experimental data on the strangeness production in all these models. In all the thermodynamically consistent models with excluded-volume correction, the value of μ_S first increases as μ_B

Table 3. Calculated thermal parameters and the freeze-out particle ratios obtained in the present model as well as in Uddin-Singh model

	Present Model	Uddin-Singh Model
λ_q	1.565	1.595
λ_s	1.06	1.08
γ_s	0.94	0.78
T (MeV)	175	200
$\bar{\Lambda}/\Lambda$	0.194	0.189
K^+/K^-	1.99	1.92
$\Lambda/(p-\bar{p})$	0.39	0.39
$\bar{\Xi}/\Xi$	0.475	0.404
$\bar{\Xi}^+/\bar{\Lambda}$	0.24	0.183
Ξ^-/Λ	0.099	0.086

increases from zero but after a certain value of μ_B, μ_S becomes almost constant [20].

We would now investigate the existence of a thermalized system in the ultra-relativistic heavy-ion collision. The purpose is to know whether there exists a set of parameters T, μ_B, μ_s and γ_s which describes all measured particle ratios and ensures strangeness neutrality in the thermalised system. It leads us to discard a model based on the uncorrelated superposition of nucleon-nucleon scattering. Recently Sollfrank et al. [18] used the EOS for the point-like hadrons in the HG and calculated the parameters λ_q, λ_s and γ_s analytically from the experimental ratios of $\bar{\Lambda}/\Lambda$, K^+/K^- and $\Lambda/(p-\bar{p})$. The temperature T was numerically deduced in their model by imposing the condition of strangeness neutrality. However, in our model because of the finite-size corrections in (11), we cannot calculate λ_q, λ_s and γ_s analytically. We first obtain the numerical values of the number-densities of baryons, and antibaryons from (11), similarly the number densities of mesons are also obtained treating them as point-like particles. Finally we pick-up only those values of the parameters which satisfy the above ratios together with the additional constraint of strangeness-neutrality (12) by the numerical parameter-search method.

It is worthwhile to investigate the chemical freeze-out conditions in heavy-ion collisions of S with S at 200 GeV/A at CERN SPS. The experimental information used for extracting the parameters are given in Table 2. We use the strangeness phase space saturation factor γ_s in order to get a good fit to the various experimental ratios simultaneously. In Table 3, we have summarized the values of the chemical freeze-out parameters T, μ_B and μ_s obtained in the present model and these are compared with those obtained in the Uddin-Singh model [26]. Here we have utilized the relation for the quark fugacity $\lambda_q = \exp(\mu_q/T)$ with $\mu_B = 3\mu_q$ and the strange quark fugacity $\lambda_s = \exp(\mu_s/T)$ where $\mu_s = -\mu_S + \mu_B/3$. The thermodynamical state of the fireball is given by the parameters $T = 175$ MeV, $\mu_B = 235$ MeV, $\mu_s = 10$ MeV and $\gamma_s = 0.94$. The corresponding values in Uddin-Singh model are $T = 200$ MeV, $\mu_B = 280$ MeV, $\mu_s = 15$ MeV and $\gamma_s = 0.78$. The most surprising result is the value of γ_s which is close to one in the present model. It means that unlike all other models, the present model indicates the saturation of the

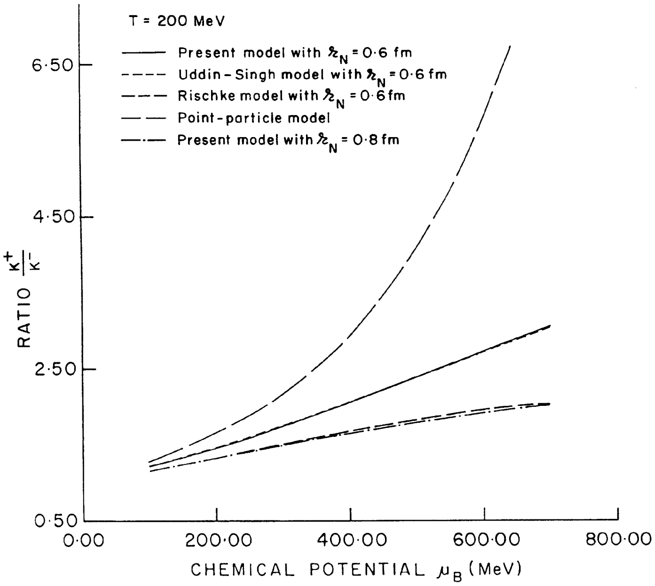

Fig. 4. The variation of ratio K^+/K^- with baryon chemical potential at $T = 200$ MeV is compared in different HG models

strangeness phase space in the hadron gas formed in S-S collision. Consequently we infer that the strangeness equilibrium is almost achieved in the heavy ion collisions according to the present EOS used here. Moreover, we notice a large difference in the values of the parameters in these models, although we find a little difference in the curves for the ratios $\bar{\Lambda}/\Lambda$ and $\bar{\Xi}/\Xi$ in different models as shown in Figs. 1 and 2. Furthermore, it is surprising that the above set of values of the parameters in our present model also account for the WA85 results at mid-rapidity for the ratios $\bar{\Xi}/\Xi$, Ξ/Λ and $\bar{\Xi}/\bar{\Lambda}$ etc. In the comparison with the WA85 results, we have not considered the kinematic cuts of the experiments and thus 4π integrated ratios agree with ratios in only a small kinematic window. We expect that the freeze-out parameters of S-S and S-W collisions should be different. It, therefore, reveals a non-trivial agreement of the freeze-out parameters for different kinds of systems likely to be formed in S-S and S-W collisions.

In order to ensure a systematic data analysis, we explore the dependence of the extracted freeze-out parameters on the assumption of the inclusion of higher resonances/particles in the HG spectrum. Sollfrank et al. [18] have noticed that the freeze-out parameters converge to the asymptotic values $\lambda_s \simeq 1$, $\lambda_q \simeq 1.6$ $\gamma_s \simeq 1$ and $T = 200$ MeV as the mass cut-off in the particle spectrum included for the analysis reaches 2 GeV. In our analysis, we have considered two cases:

(I) : HG spectrum consists of hadrons upto mass 1405 MeV, as shown in Table 1.

(II) : HG consists of hadrons upto mass 1900 MeV.

In case I, we get the freeze-out parameters of the fireball as $T = 175$ MeV, $\lambda_s = 1.06$, $\gamma_s = 0.94$ and $\lambda_q = 1.57$ while for the case II, we get $T = 220$ MeV, $\lambda_s = 1.01$, $\gamma_s = 0.99$ and $\lambda_q \simeq 1.5$. The asymptotic values of the parameters obtained in the model of Sollfrank et al. correspond to the II

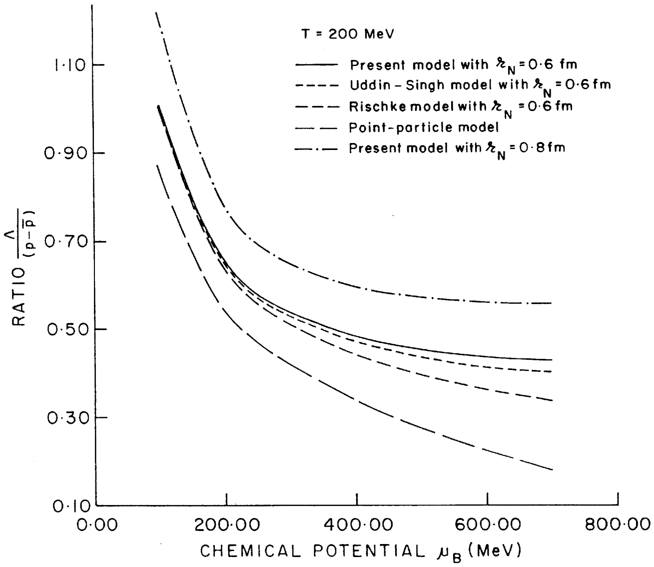

Fig. 5. The variation of ratio $\Lambda/(p-\bar{p})$ with baryon chemical potential at $T = 200$ MeV is compared in different HG models

Table 4. Total particle multiplicities obtained from the calculations in the present model as well as in the Uddin-Singh model and the model of Sollfrank et al. The quantity N_B is the net baryon content of the fireball

Hadrons	Sollfrank et al. lModel	Present lModel	Uddin-Singh lModel
K^+	12.5	12.5	12.5
K^-	6.9	6.27	6.5
Λ	7.5	7.8	7.82
$\bar{\Lambda}$	1.35	1.5	1.48
$p-\bar{p}$	18.0	20.0	20.0
h^-	65.0	55.0	63.0
η	4.9	4.9	5.54
ϕ	1.3	1.7	2.44
ω	5.5	7.0	9.63
ρ^0	6.7	5.4	6.64
π^-	54.0	46.7	54.4
p	19.4	21.8	22.16
$\bar{p}$	1.4	1.8	2.06
Δ	17.0	22.2	24.32
$\bar{\Delta}$	1.17	1.8	2.31
Ξ^-	0.73	0.80	0.67
$\bar{\Xi}^+$	0.31	0.37	0.27
N_B		53.8	53.5

case of our analysis where all the parameters are almost the same except the temperature which takes still higher value as $T = 220$ MeV. The temperature obtained in this analysis correspond to the chemical freeze-out conditions whereas the slope of the p_T-distribution yields the thermal freeze-out temperature when the particles decouple from the fireball. Since chemical freeze-out occurs earlier than the thermal freeze-out, one expects a lower value of the temperature from the p_T-distribution than that from the chemical freeze-out. However, the presence of a collective transverse flow will enhance the thermal freeze-out temperature by the blue-shifting. In case I, we get the effective thermal temperature as [27]:

$$T_{\text{eff}} = T\left[\frac{1+\beta}{1-\beta}\right]^{1/2} . \qquad (13)$$

Since $T = 175$ MeV, we can use $\beta = 0.3$ for the transverse flow and we get $T_{\text{eff}} = 238$ MeV. This compares well with a temperature $T = 228 \pm 9$ MeV as derived from the inverse slope of p_T-distribution for Λ-particles by NA36 collaborators in S-Pb collisions [27]. For $T = 220$ MeV, we get a negligible flow component and similar situation also occurs at $T = 200$ MeV as obtained by Sollfrank et al. However, at $T \geq 200$ MeV, with a negligible flow velocity, we find a consistency problem with the freeze-out since the hadronic system is so dense that we cannot treat it as a non-interacting system [27]. Moreover, we do not expect a hadron gas to survive at a temperature $T > 200$ MeV. Although the values of λ_s and γ_s reveal an equilibrium for strangeness production in both cases, the T and μ_B values are much larger in the second case and thus all kinds of hadrons are present in a very large number. So HG becomes denser than expected in the first case. Thus we find that case I with lesser number of resonances in our analysis from a thermodynamically consistent EOS yields a valid description of HG since at $T = 175$ MeV, a larger flow velocity $\beta = 0.3$ is required and the hadron-gas remains dilute at the freeze-out.

In Table 4, we have listed the multiplicities of various hadrons obtained in our thermal hadron gas models using the chemical freeze-out conditions obtained above. These values are consistent with the experimental data obtained at CERN experiments with S-S collisions. We compare our results with those obtained in Uddin-Singh model and also in the Sollfrank et al. model. Experimental value of total number of negative hadrons h^- is larger in comparison to value calculated in our model and it may be due to contributions of non-thermal origin. This is because some mesons are produced at the time of initial collision and they leave the interaction zone immediately without further interactions. Alternatively it may also be due to a generally slow absorption processes in the fireball. Similarly Table 4 also shows the net baryon number content of the fireball $N_B = 54$. This would mean that in the present S-S collision at 200 GeV/A with total number of colliding nucleons as 64, the fireball however evolves with about 54 nucleons on an average while the remaining are lost at the time of initial collisions and do not participate in the fireball formation.

The entire analysis performed here would be well outside the region of validity of the hadronic equation of state, provided the hadrons really undergo a phase transition at the temperature and chemical potentials derived in this paper. The simple way of illustrating this point will be by drawing a phase diagram which will also involve an equation of state for QGP phase as well. Usually one considers QGP as an ideal gas of quarks and gluons in a confining bag with a bag constant B. This simple picture gives $T_c = 150$ MeV at $\mu_c = 0$. However, the description of this transition region is very complicated and it requires a considerable deepening of our intuition regarding strong interaction phenomena in order to be able to identify viable simple approximation schemes. Thus we conclude that neither the ideal HG or a HG with an excluded volume approximation nor an ideal QGP approximation can work under such circumstances. For example, putting a second-order perturbative interaction term proportional to strong coupling α_s in the EOS of QGP yields a value of $T_c \sim 200$ MeV for $\alpha_s = 0.6$ at $\mu_c = 0$ [7]. Similarly the consideration of a finite-size statistical system in the EOS of HG will also reduce the freeze-out temperature further [21] by a significant amount. Therefore, the derivation of a consistency check for the thermal HG picture by drawing a phase diagram does not appear worth-while unless we get a precise method for obtaining the same. Moreover, we have assumed here that even in a hot and dense situation like the one obtained above, the HG does not undergo any phase transition so that the entire exercise as outlined above remains valid at the temperatures and chemical potentials obtained in this paper.

The very small value of μ_s and almost unit value of γ_s obtained here suggest that the predictions of a thermal hadron gas model do not differ much from those of a strangeness neutral QGP. This is a significant information which can find use in the diagnostics of QGP formation in ultra-relativistic heavy-ion collisions. If the collision history in such collisions is a thermally and chemically equilibrated hadron gas, then we observe the particle ratios when the particles decouple from such a system. Thus we conclude that the strangeness production in heavy-ion collisions does test the onset of equilibration and hence it constitutes an important tool in the study of a hot and dense hadron gas produced in such collisions.

In conclusion we have presented a model for the EOS of a dense and hot HG incorporating the exluded volume correction in a thermodynamically consistent way. The implications of the model for the strange particle production in the heavy-ion collisions are explored. Our calculations reveal that the experimental data at CERN yield a value of the freeze-out temperature $T = 175$ MeV and indicate that the state of strangeness equilibrium has almost reached in these experiments. Chemical equilibration of strangeness has a far reaching implication on the search of a QGP signal. The observed temperature is found to be much smaller than apparent temperature obtained from the inverse slope of the observed m_T-spectra which thus reveals a considerable blue-shifting effect due to collective expansion. Our value of T is smaller than what we get in pointlike hadron gas [18] and Uddin-Singh HG model. These results thus amply demonstrate the wide applicability of a statistical 'thermal fireball' model in the analysis of ultra-relativistic heavy ion collisions.

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